MULTILEVEL MODELS

Multilevel-analysis in SPSS - step by step

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Overview of a strategy

1. Data preparation (centering and standardizing)
2. Null random intercept model
3. Random intercept with Level-1 predictors
4. Adding Level-2 predictors to step 3
5. Testing random slopes level 1
6. Testing significant random slopes Level 1
7. Testing random slopes with level-2 predictors
8. Reducing parameters in var-covar
9. Testing cross-level interactions
10. Change estimation method and compare models
11. Standardizing
The basic formula

- General multilevel-model

\[ Y_{ij} = \gamma_{00} + \gamma_{10} X_{ij} + \gamma_{01} Z_j + \gamma_{11} Z_j X_{ij} + u_{1j} X_{ij} + \mu_{0j} + r_{ij} \]

All fixed parameters are in this part

= FIXED PART

All random parameters are in this part

= RANDOM PART

Overview of ML models

<table>
<thead>
<tr>
<th>Model</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>0RI</td>
<td>Null random intercept / Intercept only / Unconditional means</td>
</tr>
<tr>
<td>RI</td>
<td>Random intercept</td>
</tr>
<tr>
<td>FR</td>
<td>Fully Random</td>
</tr>
</tbody>
</table>
PHASE 1: PREPARATION

1. Data preparation

STEP 1 Data preparation

Actions:

- Look at the distribution of variables
- Check outliers
- Data centering and standardizing
- Check your centering and standardizing

(see slides data management)
PHASE 2: VARIANCE COMPONENT MODELS

2. Null random intercept model
   • PURPOSE: estimate a basic model to compare other models with

3. Random intercept with Level-1 predictors
   • PURPOSE: how much variance can be explained with your level-1 predictors?

4. Adding Level-2 predictors to step 3

PHASE 2

2. Null random intercept model
   • PURPOSE: estimate a basic model to compare other models with

3. Random intercept with level-1 variables
   • PURPOSE: how much variance can be explained with your level-1 predictors?

4. Random intercept with level 1 and level 2 variables
   • PURPOSE: how much (additional) variance can be explained with your level-2 predictors?
STEP 2 Null random intercept model

THEORETICAL: \( Y_{ij} = \gamma_{00} + \mu_{0j} + r_{ij} \)

In the example:

\( \gamma_{00} \) = Overall intercept: mean income over all countries
\( \mu_{0j} \) = Non explained differences in income between the countries
\( r_{ij} \) = Non explained differences in income between the individuals

* 2. STEP 2: Null random intercept model;
* **********************************************;

MIXED ink_centr
/PRINT = SOLUTION TESTCOV
/METHOD = REML
/FIXED = INTERCEPT
/RANDOM = INTERCEPT | SUBJECT(cntry2).
STEP 2 Null random intercept model

* 2. STEP 2: Null random intercept model;
* **********************************************;
MIXED ink_centr
/PRINT = SOLUTION TESTCOV
/METHOD = REML
/FIXED = INTERCEPT
/RANDOM = INTERCEPT | SUBJECT(cntry2).

PRINT
SOLUTION = Show the Fixed effects in the output with the standard errors
TESTCOV = Show significance tests for the variance components

METHOD
The estimation method
Used method here = Restricted Maximum Likelihood
**STEP 2 Null random intercept model**

```
* * 2. STEP 2: Null random intercept model;
**********************************************;
MI XED  ink_centr
/PRINT = SOLUTION TESTCOV
/METHOD = REML
/FIXED = INTERCEPT
/RANDOM = INTERCEPT.
```

**FIXED**
Add the fixed variables (independent variables) here.

**RANDOM**
Add the random parts here. HERE we add only the intercept (for now).

**OUTPUT FOR:**  \( Y_{ij} = \gamma_{00} + \mu_{0j} + r_{ij} \)

<table>
<thead>
<tr>
<th>Model Dimension</th>
<th>Number of Levels</th>
<th>Number of Parameters</th>
<th>Number of Subject Variables</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fixed Effects</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>Random Effects</td>
<td>Intercept</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>Residual</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>Total</td>
<td></td>
<td>4</td>
<td>2</td>
</tr>
</tbody>
</table>

- Dependent Variable: Ministm_geparamksit

**Number of parameters in the model:**

**Grouping variable:**
STEP 2 Null random intercept model

OUTPUT FOR: \( Y_{ij} = \gamma_{00} + \mu_{0j} + r_{ij} \)

Estimated mean income over all countries is -0.097

Unexplained variance (differences) on individual level equals 2.4855

Unexplained variance (differences) on country level (level 2) equals 4.0058

### Estimates of Fixed Effects

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Estimate</th>
<th>Std. Error</th>
<th>df</th>
<th>t</th>
<th>Sig.</th>
<th>Lower Bound</th>
<th>Upper Bound</th>
</tr>
</thead>
<tbody>
<tr>
<td>Intercept</td>
<td>-0.09756</td>
<td>0.03885</td>
<td>129,702</td>
<td>-2.596</td>
<td>.010</td>
<td>-0.174733</td>
<td>-0.02038</td>
</tr>
</tbody>
</table>

### Estimates of Covariance Parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Estimate</th>
<th>Std. Error</th>
<th>Wald Z</th>
<th>Sig.</th>
<th>Lower Bound</th>
<th>Upper Bound</th>
</tr>
</thead>
<tbody>
<tr>
<td>Residual</td>
<td>4.005854</td>
<td>0.03885</td>
<td>129.702</td>
<td>.000</td>
<td>3.90638</td>
<td>4.10512</td>
</tr>
<tr>
<td>Intercept (Subject = varying) variance</td>
<td>2.485587</td>
<td>0.067988</td>
<td>3.677</td>
<td>.000</td>
<td>2.348527</td>
<td>2.622647</td>
</tr>
</tbody>
</table>

### Summary table

<table>
<thead>
<tr>
<th>ORI</th>
</tr>
</thead>
<tbody>
<tr>
<td>Intercept</td>
</tr>
<tr>
<td>( \sigma^2_r )</td>
</tr>
<tr>
<td>( \sigma^2_{\mu_0} )</td>
</tr>
<tr>
<td>( \sigma^2_{\mu_1} )</td>
</tr>
</tbody>
</table>
STEP 2 Null random intercept model

- No explanation of the variance in income
- BUT a decomposition of the variance in income in 2 parts

\[ \sigma^2_{\mu_0} = \text{Inter-group variance: differences between countries} \]

\[ \sigma^2_r = \text{Intra-group variance: differences between individuals} \]

QUESTION: Are there enough differences between countries to justify a multi-level analysis?

**ANSWER:** Intra-class correlation-coefficient (RHO)

\[ \rho_1 = \frac{\sigma^2_{\mu_0}}{\sigma^2_{\mu_0} + \sigma^2_r} \]
STEP 2 Null random intercept model

Intra-class correlation-coefficient (RHO) \( (\rho_1 = \frac{\sigma^2_{ij}}{\sigma^2_{ij} + \sigma^2_{e}}) ) \)

\[
\rho_1 = \frac{2.4842}{(2.4842 + 4.0076)} = 0.383
\]

INTERPRETATION: 38.3 % of the total variance can be ascribed to level 2.

PUT DIFFERENTLY: For 38.3 %, the differences in income between countries in the ESS are due to differences between countries (richer and poorer countries). 61.7 % is due to differences in individuals within these countries (richer and poorer individuals within a country).

STEP 3 Random intercept (level 1)

THEORETICAL: \( Y_{ij} = \gamma_{00} + \gamma_{10} X_{ij} + \mu_{0j} + r_{ij} \)

In the example:

\( \gamma_{00} \) = Overall intercept: mean income over all countries
\( \mu_{0j} \) = Non explained differences in income between the countries
\( r_{ij} \) = Non explained differences in income between the individuals
\( \gamma_{10} X_{ij} \) = General slope of the independent variable X (gender or education)

QUESTION: How much of the original variance in \( \mu_{0j} \) has been explained now?
STEP 3 Random intercept (level 1)

BASIC IDEA:
Add all your level-1 predictors

The variance components in the residuals should become smaller because the level-1 predictors now take up a part of the explanation of the total variance.

QUESTION: How big is the explained part?

* 3. STEP 3: Random intercept model with level 1 predictors:

```
MIXED ink_centr BY geslacht WITH opl_centr
/PRINT = SOLUTION TESTCOV
/METHOD = REML
/FIXED = INTERCEPT geslacht opl_centr
/RANDOM = INTERCEPT | SUBJECT(cntry2).
```

Add your Level-1 independent variables
STEP 3 Random intercept (level 1)

OUTPUT FOR: \( Y_{ij} = \gamma_{00} + \gamma_{10} X_{ij} + \mu_{0j} + r_{ij} \)

**Estimates of Fixed Effects**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Estimate</th>
<th>Std. Error</th>
<th>df</th>
<th>t</th>
<th>Sig</th>
</tr>
</thead>
<tbody>
<tr>
<td>Intercept</td>
<td>-235551</td>
<td>31951</td>
<td>2140</td>
<td>-756</td>
<td>000</td>
</tr>
<tr>
<td>gender dummy (0)</td>
<td>028548</td>
<td>00000</td>
<td>33488211</td>
<td>14.772</td>
<td>0000</td>
</tr>
<tr>
<td>gender dummy (1)</td>
<td>0</td>
<td>0</td>
<td>33488211</td>
<td>14.772</td>
<td>0000</td>
</tr>
<tr>
<td>age centroid</td>
<td>401176</td>
<td>0971535</td>
<td>33469018</td>
<td>70.242</td>
<td>0000</td>
</tr>
</tbody>
</table>

- a. This parameter is set to zero because it is redundant.
- b. Dependent variable: Income in euros.

**Estimates of Covariance Parameters**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Estimate</th>
<th>Std. Error</th>
<th>Wald Z</th>
<th>Sig</th>
</tr>
</thead>
<tbody>
<tr>
<td>Residual variance</td>
<td>3443746</td>
<td>0097259</td>
<td>179445</td>
<td>000</td>
</tr>
<tr>
<td>Intercept variance</td>
<td>230986</td>
<td>059233</td>
<td>22530</td>
<td>000</td>
</tr>
</tbody>
</table>

- b. Error variance on level 1.
- c. Error variance on level 2.

The effect of gender and education on income is significant.

Even after controlling for level-1 predictors, there still are significant differences between individuals on level-1 and significant differences between the countries on level 2.
STEP 3 Random intercept (level 1)

Reduction of individual differences:

From 4.0058 to 3.4640

Reduction of differences between countries:

From 2.4855 to 2.1303

Is this enough? We need an $R^2$-measure

---

Regression-like $R^2$-measure on level 1:

$$\frac{r_{\text{Basis}} - r_{\text{Model}}}{r_{\text{Basis}}} \quad \frac{4.0058 - 3.4640}{4.0058}$$

Regression-like $R^2$-measure on level 2:

$$\frac{\mu_{\text{Basis}} - \mu_{\text{Model}}}{\mu_{\text{Basis}}} \quad \frac{2.4855 - 2.1303}{2.4855}$$

BUT: The estimation methods make this easy solution erroneous.
STEP 3 Random intercept (level 1)

No consensus yet about a good $R^2$-measure.

Correction of Bosker&Snijders (1994) – level 1:

$$(r_{\text{Basis}} + \mu_{\text{Basis}}) - (r_{\text{Model}} + \mu_{\text{Model}}) / (r_{\text{Basis}} + \mu_{\text{Basis}})$$

Correction of Bosker&Snijders (1994) – level 2:

$$(r_{\text{Basis}} + (\mu_{\text{Basis}} / n)) - (r_{\text{Model}} + (\mu_{\text{Model}} / n)) / (r_{\text{Basis}} + (\mu_{\text{Basis}} / n))$$

Whereby $n = \text{mean cluster size at level 2}$

STEP 3 Random intercept (level 1)

Correction of Bosker&Snijders (1994) – level 1:

$$(r_{\text{Basis}} + \mu_{\text{Basis}}) - (r_{\text{Model}} + \mu_{\text{Model}}) / (r_{\text{Basis}} + \mu_{\text{Basis}}) = 0.14$$

Correction of Bosker&Snijders (1994) – level 2:

$$(r_{\text{Basis}} + (\mu_{\text{Basis}} / n)) - (r_{\text{Model}} + (\mu_{\text{Model}} / n)) / (r_{\text{Basis}} + (\mu_{\text{Basis}} / n)) = 0.14$$
STEP 4 Random intercept (level 1+2)

THEORETICAL: $Y_{ij} = \gamma_{00} + \gamma_{10} X_{ij} + \gamma_{01} Z_j + \mu_{0j} + r_{ij}$

In the example:

$\gamma_{00}$ = Overall intercept: mean income over all countries  
$\mu_{0j}$ = Non explained differences in income between the countries  
$r_{ij}$ = Non explained differences in income between the individuals  
$\gamma_{10} X_{ij}$ = General slope of the independent variable X (gender or education)  
$\gamma_{01} Z_j$ = Direct effect of country variables on income (BNP or religion)

QUESTION: How much variance in $\mu_{0j}$ from step 3 is now further explained?

STEP 4 Random intercept (level 1+2)

BASIC IDEA:      
Look how much the level 2 predictors can explain (as a group)
STEP 4 Random intercept (level 1+2)

* 4. STEP 4: Random intercept model with level 1 and level 2 predictors;
* *********************************************************

MIXED ink_centr BY geslacht WITH opl_centr bnp_centr bnppps_centr romkath_centr relig_centr
/PRINT = SOLUTION TESTCOV
/METHOD = REML
/FIXED = INTERCEPT geslacht opl_centr bnp_centr bnppps_centr romkath_centr relig_centr
/RANDOM = INTERCEPT | SUBJECT(cntry2).

OUTPUT FOR: $Y_{ij} = \gamma_00 + \gamma_{10} X_{ij} + \gamma_{01} Z_j + \mu_{0j} + r_{ij}$

- Overall intercept
- Coefficients on level-1
- Coefficients on level-2
- Error-variance on level 1
- Error-variance on level 2
STEP 4 Random intercept (level 1+2)

OUTPUT FOR: \( Y_{ij} = \gamma_{00} + \gamma_{10} X_{ij} + \gamma_{01} Z_{ij} + \mu_{0j} + r_{ij} \)

### Estimates of Fixed Effects

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Estimate</th>
<th>Std. Error</th>
<th>df</th>
<th>t</th>
<th>Sig</th>
<th>95% Confidence Interval</th>
</tr>
</thead>
<tbody>
<tr>
<td>Intercept</td>
<td>-0.258282</td>
<td>0.1605</td>
<td>15.106</td>
<td>-1.645</td>
<td>.109</td>
<td></td>
</tr>
<tr>
<td>gesucht.01</td>
<td>0.000006</td>
<td>0.021421</td>
<td>28931.547</td>
<td>13.415</td>
<td>.000</td>
<td></td>
</tr>
<tr>
<td>gesucht.1.01</td>
<td>0</td>
<td>0</td>
<td>28931.547</td>
<td>13.415</td>
<td>.000</td>
<td></td>
</tr>
<tr>
<td>cat_L ALERT</td>
<td>0.003286</td>
<td>0.007097</td>
<td>29944.363</td>
<td>47.029</td>
<td>.000</td>
<td>0.002848 (\ldots) 0.017044</td>
</tr>
<tr>
<td>cat_L ALERT</td>
<td>2.182387</td>
<td>0.045145</td>
<td>14.973</td>
<td>-2.139</td>
<td>.035</td>
<td>-0.002173 (\ldots) 0.002530</td>
</tr>
<tr>
<td>scope_cent</td>
<td>0.029725</td>
<td>0.004361</td>
<td>15.025</td>
<td>6.312</td>
<td>.000</td>
<td>0.018231 (\ldots) 0.04019</td>
</tr>
<tr>
<td>scope_cent</td>
<td>-0.105921</td>
<td>0.045145</td>
<td>14.973</td>
<td>-2.319</td>
<td>.035</td>
<td>-0.002173 (\ldots) 0.002530</td>
</tr>
<tr>
<td>reg_cen</td>
<td>0.155499</td>
<td>0.017343</td>
<td>15.000</td>
<td>0.000</td>
<td>.999</td>
<td>-0.006873 (\ldots) 0.006771</td>
</tr>
</tbody>
</table>

* a. This parameter is set to zero because it is redundant.

BNPPPS and % Roman Catholics are significant, BNP and Religiosity not.

### Even after controlling for level-2 predictors

There still are significant differences between individuals on level-1 and significant differences between the countries on level 2.

### Type III Tests of Fixed Effects

<table>
<thead>
<tr>
<th>Source</th>
<th>Num df</th>
<th>Denom df</th>
<th>F</th>
<th>Sig</th>
</tr>
</thead>
<tbody>
<tr>
<td>Model</td>
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<td>14.940</td>
<td>.459</td>
<td>.510</td>
</tr>
<tr>
<td>gesucht.01</td>
<td>1</td>
<td>28931.547</td>
<td>179.955</td>
<td>.000</td>
</tr>
<tr>
<td>gesucht.1.01</td>
<td>1</td>
<td>28931.547</td>
<td>4492.123</td>
<td>.000</td>
</tr>
<tr>
<td>BNP_PROP</td>
<td>0</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>reg_cen</td>
<td>1</td>
<td>15.025</td>
<td>39.837</td>
<td>.000</td>
</tr>
<tr>
<td>reg_cen.01</td>
<td>1</td>
<td>14.973</td>
<td>5.378</td>
<td>.035</td>
</tr>
<tr>
<td>reg_cen.00</td>
<td>1</td>
<td>15.000</td>
<td>0.001</td>
<td>.999</td>
</tr>
</tbody>
</table>

* a. Dependent variable: Incomes received

Problem with BNP?
STEP 4 Random intercept (level 1+2)

Reduction of individual differences:

3 vs 2  From 3.464 to 3.476
3 vs 1  From 4.006 to 3.476

Reduction of country differences:

3 vs 2  From 2.130 to 0.552
3 vs 1  From 2.486 to 0.552

Correction of Bosker&Snijders (1994) – level 1:

Pseudo – $R^2 = 0.38$

Correction of Bosker&Snijders (1994) – level 2:

Pseudo – $R^2 = 0.78$
STEP 4 Random intercept (level 1+2)

Preliminary conclusions:

1. The error-variance on level 2 is only significant to the 0.006 level.

2. Two level-2 variables do not seem to work in this model: BNP and Religiosity. We remove them from the model when they are less important from a theoretical perspective.

3. Our model strongly explains differences on level 2. But also on the individual level the explanatory power of the independent variables is satisfactory.

PHASE 3: testing random slopes

5. Testing random slopes level 1
6. Testing significant random slopes Level 1
7. Testing random slopes with level-2 predictors
8. Reducing the parameters in var-covar
PHASE 3

5. Testing random slopes level 1
   • PURPOSE: looking which level-1 predictor has significant slopes (without the level-2 predictors).

6. Testing significant random slopes Level 1
   • PURPOSE: joining all significant random slopes from step 5 in one model

7. Testing random slopes with level-2 predictors
   • PURPOSE: adding the level 2 predictors back to model, including the random slopes from step 6

8. Reducing parameters in var-covar
   • PURPOSE: simplifying the model (removing non-significant co-variances) in order to ease the estimation of the model.

STEP 5 Testing random slopes level 1

THEORETICAL : \( y_{ij} = \gamma_{00} + \gamma_{10} x_{ij} + \mu_{ij} x_{ij} + \nu_{0j} + r_{ij} \)

In the example:
\( \gamma_{00} \) = Overall intercept: mean income over all countries
\( \mu_{ij} \) = Non explained differences in income between the countries
\( r_{ij} \) = Non explained differences in income between the individuals
\( \gamma_{10} x_{ij} \) = General slope of the independent variable X (gender or education)
\( \mu_{ij} x_{ij} \) = Differences in slopes (effect) of variable X (gender or education) between the countries

REMARK: No longer country differences in this model!
REMARK: Model is estimated for each independent variable on level 1 separately
QUESTION: Are there significant differences in the effect of X between the countries?
STEP 5 Testing random slopes level 1

BASIC IDEA:
Until now we did not allow for differences in slopes of the independent variables at level 1.

FOR EACH INDEPENDENT, we will test if there are significant differences in slopes at level-2.

***********************************************************************************;
MIXED ink_centr BY geslacht WITH opl_centr
/PRINT = SOLUTION TESTCOV
/METHOD = REML
/FIXED = INTERCEPT geslacht opl_centr
/RANDOM = INTERCEPT opl_centr | SUBJECT(cntry2) COVTYPE(UN).

COVTYPE should be "Unstructured".
We make eduction RANDOM here.
STEP 5 Testing random slopes level 1

OUTPUT FOR: \( Y_{ij} = \gamma_{00} + \gamma_{10} X_{ij} + u_{0j} + u_{1j} X_{ij} + r_{ij} \)

Estimation of Fixed Effects:\

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Estimate</th>
<th>Std. Error</th>
<th>t</th>
<th>Sig.</th>
<th>Lower Bound</th>
<th>Upper Bound</th>
</tr>
</thead>
<tbody>
<tr>
<td>Intercept</td>
<td>-2.12331</td>
<td>.102238</td>
<td>20.544</td>
<td>.000</td>
<td>-.271000</td>
<td>-.175600</td>
</tr>
<tr>
<td>( \beta_{10} )</td>
<td>.4355681</td>
<td>.047055</td>
<td>44.095</td>
<td>.000</td>
<td>.341750</td>
<td>.529388</td>
</tr>
<tr>
<td>( \sigma_{\mu}^2 )</td>
<td>3.21123</td>
<td>.003258</td>
<td>991.003</td>
<td>.000</td>
<td>2.87944</td>
<td>3.54320</td>
</tr>
</tbody>
</table>

A. This parameter is set to zero because it redundant.
B. Dependent Variable: Innom one corrected.

Error-variance on level 2

Estimation of Covariance Parameters:\

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Estimate</th>
<th>Std. Error</th>
<th>t</th>
<th>Sig.</th>
<th>Lower Bound</th>
<th>Upper Bound</th>
</tr>
</thead>
<tbody>
<tr>
<td>Residual</td>
<td>.3123173</td>
<td>.0261238</td>
<td>112.317</td>
<td>.000</td>
<td>.267000</td>
<td>.367600</td>
</tr>
</tbody>
</table>

Error-variance on level 1

Structure of the variance-covariance matrix

Theoretical: \( \begin{bmatrix} \mu_{ij} \\ \mu_{ij} \end{bmatrix} \sim N(0, \Omega_p): \Omega_p = \begin{bmatrix} \sigma_{\mu}^2 & \sigma_{\mu u} \\ \sigma_{\mu u} & \sigma_{u}^2 \end{bmatrix} \)

In SPSS:

\[
\begin{bmatrix}
UM & UN1 \\
UN2 & UN2
\end{bmatrix} = \begin{bmatrix} 2.1123 & .003258 \\
.003258 & 0.01991
\end{bmatrix}
\]

Estimation of Covariance Parameters:\

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Estimate</th>
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<td>112.317</td>
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<td>.267000</td>
<td>.367600</td>
</tr>
</tbody>
</table>

A. Dependent Variable: Innom one corrected.
STEP 5 Testing random slopes level 1

Structure of the variance-covariance matrix

In SPSS:

\[
\begin{pmatrix}
UN_{11} & 2.1123 \\
UN_{21} & 0.03258 \\
UN_{22} & 0.00199
\end{pmatrix}
\]

The random slope is significant on the 0.01 level.

**MEANING:** there is a significant different effect of education between the countries.

---

STEP 5 Testing random slopes level 1

*STEP 5: Testing the significance of the random slopes step-by-step.*

```
MIXED ink_centr BY geslacht WITH opl_centr
/PRINT = SOLUTION TESTCOV
/METHOD = REML
/FIXED = INTERCEPT geslacht opl_centr
/RANDOM = INTERCEPT(geslacht) SUBJECT(cntry2) COVTYPE(UN).
```

Gender is made RANDOM here.
STEP 5 Testing random slopes level 1

OUTPUT FOR: Y_{ij} = \gamma_0 + \gamma_1 X_{ij} + \mu_1 X_{ij} + \mu_0 + r_{ij}

Estimation of Fixed Effects 1:

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Estimate</th>
<th>Std. Error</th>
<th>t</th>
<th>Sig.</th>
<th>Lower Bound</th>
<th>Upper Bound</th>
</tr>
</thead>
<tbody>
<tr>
<td>Intercept</td>
<td>2.521593</td>
<td>0.589893</td>
<td>-4.3</td>
<td>.000</td>
<td>1.332870</td>
<td>3.700317</td>
</tr>
<tr>
<td>\gamma_1</td>
<td>0.583208</td>
<td>0.317070</td>
<td>1.8</td>
<td>.071</td>
<td>-0.114660</td>
<td>1.280138</td>
</tr>
<tr>
<td>\mu_1</td>
<td>0</td>
<td>0</td>
<td></td>
<td>.000</td>
<td>-4.025672</td>
<td>4.025672</td>
</tr>
<tr>
<td>\mu_0</td>
<td>0</td>
<td>0</td>
<td></td>
<td>.000</td>
<td>-4.025672</td>
<td>4.025672</td>
</tr>
</tbody>
</table>

Estimation of Covariance Parameters 2:

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Estimate</th>
<th>Std. Error</th>
<th>t</th>
<th>Sig.</th>
<th>Lower Bound</th>
<th>Upper Bound</th>
</tr>
</thead>
</table>

Overall intercept

Coefficients on level-1

Error-variance on level 1

Error-variance on level 2

Structure of the variance-covariance matrix

Theoretical:

\[ \begin{bmatrix} \mu_0 \\ \mu_1 \end{bmatrix} \sim N(0, \Omega), \Omega = \begin{bmatrix} \sigma_{\mu_0}^2 & \sigma_{\mu_0\mu_1} \\ \sigma_{\mu_1\mu_0} & \sigma_{\mu_1}^2 \end{bmatrix} \]

In SPSS:

\[ \begin{bmatrix} U1M1 \\ U1N2,1 \\ U1N2,2 \end{bmatrix} = \begin{bmatrix} 1.144 \\ 0.4728 \\ 0.6207 \end{bmatrix} \]

Estimation of Covariance Parameters 3:

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Estimate</th>
<th>Std. Error</th>
<th>t</th>
<th>Sig.</th>
<th>Lower Bound</th>
<th>Upper Bound</th>
</tr>
</thead>
</table>

Overall intercept

Coefficients on level-1

Error-variance on level 1

Error-variance on level 2

...
**STEP 5 Testing random slopes level 1**

Evaluation of the random slope of gender

In SPSS:

\[
\begin{bmatrix}
U_{N1} \\
U_{N21} \\
U_{N22}
\end{bmatrix} = \begin{bmatrix}
2.1725 \\
-0.04622 \\
0.0097
\end{bmatrix}
\]

The random slope of the dichotomous variable gender can not be tested. If you include “gender” as a continuous variable in the model, this random slope turns out to be significant.

---

**STEP 5 Testing random slopes level 1**

Conclusion:

- Education and gender have significant slopes between the countries.
- Gender is only estimable when you include the variable as a continuous measure.

BUT: the effect of gender is unclear.

In this example I will keep gender in the model with a random slope (only for educational purposes).
STEP 6 Testing random slopes level 1+2

THEORETICAL: \( Y_{ij} = \gamma_{00} + \gamma_{10} X_{ij} + \mu_{1j} X_{ij} + \mu_{0j} + r_{ij} \)

- \( \gamma_{00} \): Overall intercept: mean income over all countries
- \( \mu_{0j} \): Non explained differences in income between the countries
- \( r_{ij} \): Non explained differences in income between the individuals
- \( \gamma_{10} X_{ij} \): General slope of the independent variable \( X \) (gender or education)
- \( \mu_{1j} X_{ij} \): Differences in slopes (effect) of variable \( X \) (gender or education) between the countries

Theoretically this is the same model as in step 5 but now we include ALL significant random slopes in the model.

* STEP 6: Including all level-1 variables with significant random slopes.

```
MIXED ink_centr BY geslacht WITH opl_centr
   /PRINT = SOLUTION TESTCOV
   /METHOD = REML
   /FIXED = INTERCEPT geslacht opl_centr
   /RANDOM = INTERCEPT opl_centr geslacht | SUBJECT(cntry2) COVTYPE(UN).
```
STEP 6 Testing random slopes level 1+2

OUTPUT FOR: $Y_{ij} = \gamma_{00} + \gamma_{10} X_{ij} + \mu_{ij} X_{ij} + \mu_{0j} + r_{ij}$

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Estimate</th>
<th>Std. Error</th>
<th>t-Value</th>
<th>df</th>
<th>Lower Bound</th>
<th>Upper Bound</th>
</tr>
</thead>
<tbody>
<tr>
<td>Residual</td>
<td>3.418664</td>
<td>0.114551</td>
<td>30.766</td>
<td>290</td>
<td>3.186</td>
<td>3.651</td>
</tr>
<tr>
<td>UN(1,1)</td>
<td>0.000001</td>
<td>0.000000</td>
<td>0.000</td>
<td>290</td>
<td>0.000</td>
<td>0.000</td>
</tr>
<tr>
<td>UN(2,2)</td>
<td>0.000001</td>
<td>0.000000</td>
<td>0.000</td>
<td>290</td>
<td>0.000</td>
<td>0.000</td>
</tr>
<tr>
<td>UN(3,3)</td>
<td>0.000001</td>
<td>0.000000</td>
<td>0.000</td>
<td>290</td>
<td>0.000</td>
<td>0.000</td>
</tr>
</tbody>
</table>

Equal digits are error-variances:
- UN(1,1) Intercept
- UN(2,2) Error-variance of education
- UN(3,3) Error-variance of gender

So: Gender NOT random in the model!
STEP 7 Testing random slopes with level-2 predictors

**THEORETICAL:** \( Y_{ij} = \gamma_{00} + \gamma_{10} X_{ij} + \gamma_{01} Z_{j} + \mu_{ij} X_{ij} + \mu_{0j} + r_{ij} \)

- \( \gamma_{00} \) = Overall intercept: mean income over all countries
- \( \mu_{0j} \) = Non explained differences in income between the countries
- \( r_{ij} \) = Non explained differences in income between the individuals
- \( \gamma_{10} X_{ij} \) = General slope of the independent variable X (gender or education)
- \( \mu_{ij} X_{ij} \) = Differences in slopes (effect) of variable X (gender or education) between the countries
- \( \gamma_{01} Z_{j} \) = Direct effect of country variables on income (GNP or religion)

**BASIC IDEA:**
We now check which influence the level-2 predictors have on income in a model with random slopes (= step 6).
**STEP 7 Testing random slopes with level-2 predictors**

* STEP 7: Including all level-2 variables in the model of step 6

**MIXED** `ink_centr BY geslacht WITH opl_centr bnppps_centr romkath_centr` /PRINT = SOLUTION TESTCOV /METHOD = REML /FIXED = INTERCEPT geslacht opl_centr bnppps_centr romkath_centr /RANDOM = INTERCEPT opl_centr | SUBJECT(cntry2) COVTYPE(UN).

**OUTPUT FOR:**

\[ Y_{ij} = \beta_0 + \beta_1 X_{ij} + \beta_2 Z_j + \beta_3 X_j + \beta_4 O_{ij} + r_{ij} \]

- Overall intercept
- Coefficients on level-1
- Coefficients on level-2

**Estimates of Fixed Effects**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Estimate</th>
<th>SE</th>
<th>z</th>
<th>p-value</th>
<th>95% CI</th>
</tr>
</thead>
<tbody>
<tr>
<td>Intercept</td>
<td>-4.4448</td>
<td>1.208</td>
<td>-3.68</td>
<td>.002</td>
<td>(-6.8595, -2.0297)</td>
</tr>
<tr>
<td>geslacht</td>
<td>0.01515</td>
<td>0.021</td>
<td>0.73</td>
<td>.463</td>
<td>(-0.0276, 0.0578)</td>
</tr>
<tr>
<td>opl_centr</td>
<td>0.04227</td>
<td>0.045</td>
<td>0.94</td>
<td>.347</td>
<td>(-0.0489, 0.1333)</td>
</tr>
<tr>
<td>bnppps_centr</td>
<td>0.04157</td>
<td>0.043</td>
<td>0.97</td>
<td>.332</td>
<td>(-0.0489, 0.1319)</td>
</tr>
<tr>
<td>romkath_centr</td>
<td>0.04227</td>
<td>0.045</td>
<td>0.94</td>
<td>.347</td>
<td>(-0.0489, 0.1333)</td>
</tr>
</tbody>
</table>

- a. This parameter is set to zero because it is redundant
- b. Dependent Variable: `ink_centr`.

**Estimates of Covariance Parameters**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Estimate</th>
<th>SE</th>
<th>z</th>
<th>p-value</th>
<th>95% CI</th>
</tr>
</thead>
<tbody>
<tr>
<td>Residuals</td>
<td>2.74018</td>
<td>0.252</td>
<td>10.86</td>
<td>&lt;.001</td>
<td>(1.2842, 4.2972)</td>
</tr>
<tr>
<td>Residuals (subject: cntry2)</td>
<td>0.01758</td>
<td>0.007</td>
<td>2.43</td>
<td>.015</td>
<td>(0.0036, 0.0316)</td>
</tr>
<tr>
<td>Residuals (subject: cntry2)</td>
<td>0.01758</td>
<td>0.007</td>
<td>2.43</td>
<td>.015</td>
<td>(0.0036, 0.0316)</td>
</tr>
</tbody>
</table>


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STEP 7 Testing random slopes with level-2 predictors

OUTPUT FOR: \( Y_{ij} = \gamma_{00} + \gamma_{10} X_{ij} + \gamma_{01} Z_j + \mu_{1j} X_{ij} + \mu_{0j} + \epsilon_{ij} \)

\( \epsilon_{ij} \sim N(0, \sigma^2) \)

Estimated Fixed Effects:

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Estimate</th>
<th>Std. Error</th>
<th>t</th>
<th>Sig.</th>
<th>Lower Bound</th>
<th>Upper Bound</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \gamma_{00} )</td>
<td>9.345</td>
<td>1.230</td>
<td>7.56</td>
<td>0.000</td>
<td>6.913</td>
<td>11.777</td>
</tr>
<tr>
<td>( \gamma_{10} )</td>
<td>-0.123</td>
<td>0.025</td>
<td>-4.92</td>
<td>0.000</td>
<td>-0.173</td>
<td>-0.073</td>
</tr>
<tr>
<td>( \gamma_{01} )</td>
<td>0.123</td>
<td>0.025</td>
<td>4.92</td>
<td>0.000</td>
<td>0.073</td>
<td>0.173</td>
</tr>
</tbody>
</table>

- \( \gamma_{00} \): Overall intercept: mean income over all countries
- \( \gamma_{10} \): General slope of the independent variable X (gender or education)
- \( \gamma_{01} \): Direct effect of country variables on income (GNP or religion)
- \( \mu_{1j} \): Differences in slopes (effect) of variable X (gender or education) between the countries
- \( \epsilon_{ij} \): Non explained differences in income between the individuals

Both the predictors on level 1 as on level 2 are significant.

Theoretical: \( Y_{ij} = \gamma_{00} + \gamma_{10} X_{ij} + \gamma_{01} Z_j + \mu_{1j} X_{ij} + \mu_{0j} + \epsilon_{ij} \)

STEP 8 Simplifying the model

\( \gamma_{00} \) = Overall intercept: mean income over all countries

\( \mu_{ij} \) = Non explained differences in income between the countries

\( \epsilon_{ij} \) = Non explained differences in income between the individuals

\( \gamma_{10} X_{ij} \) = General slope of the independent variable X (gender or education)

\( \mu_{ij} X_{ij} \) = Differences in slopes (effect) of variable X (gender or education) between the countries

\( \gamma_{01} Z_j \) = Direct effect of country variables on income (GNP or religion)

We now look at the variance-covariance matrix
STEP 8 Simplifying the model

Differences in intercept and slope between units of level 2.

\[ Y_{ij} = \gamma_{00} + \gamma_{10} X_{ij} + \mu_{1j} X_{ij} + \mu_{0j} + r_{ij} \]

These are the conditions:

\[ r_{ij} \sim N(0, \sigma^2) \]

\[ \begin{bmatrix} \mu_0 \\ \mu_1 \end{bmatrix} \sim N(0, \Omega) \]

\[ \Omega = \begin{bmatrix} \sigma_{\mu_0}^2 & \sigma_{\mu_0\mu_1} \\ \sigma_{\mu_0\mu_1} & \sigma_{\mu_1}^2 \end{bmatrix} \]

This is the covariance between the random intercepts and the random slopes for variable \( X_{ij} \).

If this covariance is significant (\( \neq 0 \)), then it means that the changes in intercepts are systematically correlated to the changes in slopes.

<table>
<thead>
<tr>
<th>Model</th>
<th>intercept</th>
<th>Slopes</th>
<th>Covariance</th>
</tr>
</thead>
<tbody>
<tr>
<td>FR</td>
<td>many</td>
<td>many</td>
<td>Negative</td>
</tr>
<tr>
<td>FR</td>
<td>many</td>
<td>many</td>
<td>Positive</td>
</tr>
<tr>
<td>FR</td>
<td>many</td>
<td>many</td>
<td>Not significant (0)</td>
</tr>
</tbody>
</table>
**STEP 8 Simplifying the model**

With 1 independent X-variable on level 1

\[
\begin{bmatrix}
\mu_0 \\
\mu_1 \\
\end{bmatrix} \sim \mathcal{N}(0,\Omega_p) ; \Omega_p = \begin{bmatrix}
\sigma^2_{\mu_0} & \sigma^2_{\mu_{01}} \\
\sigma^2_{\mu_{01}} & \sigma^2_{\mu_1} \\
\end{bmatrix}
\]

Covariance between the random intercepts and the random slopes for variable \(X_i\).

With 2 independent X-variables on level 1

\[
\begin{bmatrix}
\mu_0 \\
\mu_1 \\
\mu_{ij} \\
\end{bmatrix} \sim \mathcal{N}(0,\Omega_p) ; \Omega_p = \begin{bmatrix}
\sigma^2_{\mu_0} & \sigma^2_{\mu_{01}} & \sigma^2_{\mu_{0j}} \\
\sigma^2_{\mu_{01}} & \sigma^2_{\mu_1} & \sigma^2_{\mu_{1j}} \\
\sigma^2_{\mu_{0j}} & \sigma^2_{\mu_{1j}} & \sigma^2_{\mu_{jj}} \\
\end{bmatrix}
\]

2 independents = 3 covariances

With 3 independent X-variables on level 1

\[
\begin{bmatrix}
\mu_0 \\
\mu_1 \\
\mu_{ij} \\
\mu_{ijk} \\
\end{bmatrix} \sim \mathcal{N}(0,\Omega_p) ; \Omega_p = \begin{bmatrix}
\sigma^2_{\mu_0} & \sigma^2_{\mu_{01}} & \sigma^2_{\mu_{0j}} & \sigma^2_{\mu_{0k}} \\
\sigma^2_{\mu_{01}} & \sigma^2_{\mu_1} & \sigma^2_{\mu_{1j}} & \sigma^2_{\mu_{1k}} \\
\sigma^2_{\mu_{0j}} & \sigma^2_{\mu_{1j}} & \sigma^2_{\mu_{jj}} & \sigma^2_{\mu_{jk}} \\
\sigma^2_{\mu_{0k}} & \sigma^2_{\mu_{1k}} & \sigma^2_{\mu_{jk}} & \sigma^2_{\mu_{kk}} \\
\end{bmatrix}
\]

3 independents = 6 covariances

With random slopes models, the number of parameters to be estimated increases very rapidly.
STEP 8 Simplifying the model

With 3 independent X-variables on level 1

$\begin{bmatrix}
\mu_3 \\
\mu_2 \\
\mu_1
\end{bmatrix} \sim N(0, \Omega_{\mu})$, $\Omega_{\mu} = \begin{bmatrix}
\sigma_{\mu}^2 & \sigma_{\mu_3 \mu_1} & \sigma_{\mu_3 \mu_2} \\
\sigma_{\mu_3 \mu_1} & \sigma_{\mu_2}^2 & \sigma_{\mu_2 \mu_1} \\
\sigma_{\mu_3 \mu_2} & \sigma_{\mu_2 \mu_1} & \sigma_{\mu_1}^2
\end{bmatrix}$

Non-significant covariances = The covariances is not significantly different from 0.

SO: We might as well fix these to 0.

CONSEQUENCE: Each fixed covariance is one parameter less to be estimated.

SPSS does not allow to fix each separate covariance to 0. BUT, there is an option to fix all co-variances to 0 in one time.
**STEP 8 Simplifying the model**

With 3 independent X-variables on level 1

\[
\begin{bmatrix}
\mu_0 \\
\mu_1 \\
\mu_2 \\
\mu_3
\end{bmatrix}
\sim N(0, \Omega_p) ; \Omega_p = \begin{bmatrix}
\sigma_{\mu_0}^2 & \sigma_{\mu_0\mu_1} & \sigma_{\mu_0\mu_2} & \sigma_{\mu_0\mu_3} \\
\sigma_{\mu_0\mu_1} & \sigma_{\mu_1}^2 & \sigma_{\mu_1\mu_2} & \sigma_{\mu_1\mu_3} \\
\sigma_{\mu_0\mu_2} & \sigma_{\mu_1\mu_2} & \sigma_{\mu_2}^2 & \sigma_{\mu_2\mu_3} \\
\sigma_{\mu_0\mu_3} & \sigma_{\mu_1\mu_3} & \sigma_{\mu_2\mu_3} & \sigma_{\mu_3}^2
\end{bmatrix}
\]

**COVTYPE(UN)**

Without COVTYPE

**STEP 8 Simplifying the model**

Look at the (significance of) the co-variances in the output of step 7

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Estimate</th>
<th>Std.Err</th>
<th>Wald</th>
<th>Sig.</th>
<th>95% Confidence Interval</th>
<th>Lower Bound</th>
<th>Upper Bound</th>
</tr>
</thead>
<tbody>
<tr>
<td>Intercept</td>
<td>5.42018</td>
<td>0.05460</td>
<td>122.196</td>
<td>.000</td>
<td>5.34592</td>
<td>5.49440</td>
<td></td>
</tr>
<tr>
<td>Int(1)</td>
<td>5.47432</td>
<td>0.05444</td>
<td>123.350</td>
<td>.000</td>
<td>5.36484</td>
<td>5.58380</td>
<td></td>
</tr>
<tr>
<td>Int(1)</td>
<td>0.24917</td>
<td>0.17663</td>
<td>1.310</td>
<td>.250</td>
<td>-0.04901</td>
<td>0.54735</td>
<td></td>
</tr>
<tr>
<td>Int(1)</td>
<td>-0.01311</td>
<td>0.17663</td>
<td>0.000</td>
<td>.999</td>
<td>-0.36314</td>
<td>0.33692</td>
<td></td>
</tr>
</tbody>
</table>

a. Dependent Variable: Time (mean centre).

Consequence: Covariance is NOT significant.

**Consequence:** Estimate the model in STEP 7 again **without** the COVTYPE-option.
STEP 8 Simplifying the model

-STEP 8: Reduction of parameters in var-covar matrix

**MIXED** ink_centr BY geslacht WITH opl_centr bnppps_centr romkath_centr
/PRINT = SOLUTION TESTCOV
/METHOD = REML
/FIXED = INTERCEPT geslacht opl_centr bnppps_centr romkath_centr
/RANDOM = INTERCEPT opl_centr | SUBJECT(cntry2).

The co-variances are fixed at 0: there is no estimation of co-variances any more. Only the variances are given in the output.
PHASE 4: Refining and finishing

9. Testing cross-level interactions
   • PURPOSE: checking if level-2 predictors correlate with level-1 predictors

10. Change the estimation method and compare your models
   • PURPOSE: Compare models with Full Maximum Likelihood

11. Standardizing
   • PURPOSE: Estimating the end model again with standardized variables to check the relative influence
STEP 9 Testing cross-level interactions

**THEORETICAL:** \( Y_{ij} = \gamma_{00} + \gamma_{10} X_{ij} + \gamma_{01} Z_{j} + \gamma_{01} Z_{j} X_{ij} + \mu_{ij} X_{ij} + \mu_{0j} + r_{ij} \)

- \( \gamma_{00} \) = Overall intercept: mean income over all countries
- \( \mu_{0j} \) = Non explained differences in income between the countries
- \( r_{ij} \) = Non explained differences in income between the individuals
- \( \gamma_{10} X_{ij} \) = General slope of the independent variable X (gender or education)
- \( \mu_{ij} X_{ij} \) = Differences in slopes (effect) of variable X (gender or education) between the countries
- \( \gamma_{01} Z_{j} \) = Direct effect of country variables on income (BNP or religion)
- \( \gamma_{01} Z_{j} X_{ij} \) = Interaction of country variables and individual variables on income

What is an interaction effect?

A (significant) interaction effect shows how the effect of one independent variable changes *within* the levels of another independent variable.
STEP 9 Testing cross-level interactions

- In regular OLS regression

EG. Interaction between gender and education:

How does the effect of education on income change between boys and girls?

- In multilevel regression

EG. Interaction between education and GDP:

How does the effect of education on income change for countries with a different GDP?

TWO BASIC PRINCIPLES

1. When an interaction effect is significant, the main effects NEED TO BE included in the model.

EG:
When the interaction effect between education and GDP_PPS is included
THEN you also need the main effect of education and the main effect of GDP_PPS in the model.
2. When an interaction effect is present in a model, the main effects have a different meaning.

Meaning *without* interaction-effect
- *Education*: the change in income when the educational level rises with one unit
- *BNP_PPS*: the change in income when BNP_PPS rises with one unit
STEP 9 Testing cross-level interactions

** ** STEP 9: Testing cross-level interactions

```
MIXED ink_centr BY geslacht WITH opl_centr bnppps_centr romkath_centr
/PRINT = SOLUTION TESTCOV
/METHOD = REML
/FIXED = INTERCEPT geslacht opl_centr bnppps_centr romkath_centr
opl_centr*bnppps_centr
/RANDOM = INTERCEPT opl_centr | SUBJECT(cntry2).
```

Specifying an interaction effect = repeating both variables with * between both.

---

### Estimators of Fixed Effects

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Estimate</th>
<th>Std. Error</th>
<th>df</th>
<th>t</th>
<th>Sig.</th>
<th>95% Confidence Interval</th>
</tr>
</thead>
<tbody>
<tr>
<td>Intercept</td>
<td>-2.23256</td>
<td>.171052</td>
<td>17.000</td>
<td>-1.476</td>
<td>.168</td>
<td>(-3.6750, 0.2098)</td>
</tr>
<tr>
<td>geslacht(0.0)</td>
<td>.20124</td>
<td>.021816</td>
<td>2992.404</td>
<td>9.718</td>
<td>.000</td>
<td>(.1606, .2419)</td>
</tr>
<tr>
<td>geslacht(1.0)</td>
<td>.045044</td>
<td>.036775</td>
<td>16.968</td>
<td>.618</td>
<td>.546</td>
<td>(-.0063, .1964)</td>
</tr>
<tr>
<td>opl_centr</td>
<td>.519253</td>
<td>.049054</td>
<td>18.167</td>
<td>18.656</td>
<td>.000</td>
<td>(.4228, .6157)</td>
</tr>
<tr>
<td>bnppps_centr</td>
<td>.458918</td>
<td>.041556</td>
<td>16.040</td>
<td>-2.608</td>
<td>.009</td>
<td>(-.1021, .0020)</td>
</tr>
<tr>
<td>romkath_centr</td>
<td>-.064499</td>
<td>.049054</td>
<td>18.167</td>
<td>.618</td>
<td>.546</td>
<td>(-.1964, .0670)</td>
</tr>
<tr>
<td>opl_centr*bnppps_centr</td>
<td>-1.151169</td>
<td>.000000</td>
<td>18.167</td>
<td>.000</td>
<td>.546</td>
<td>(-1.151169, .000000)</td>
</tr>
</tbody>
</table>

- This parameter is set to zero because it is redundant.
- Dependent Variable: lntransformed

Interaction effect (not significant)
STEP 9 Testing cross-level interactions

How to interpret the interaction effect?
(if it had been significant)

1. Interpret the main effects

2. Choose an interpretation perspective

3. Write the regression equation for one of the independent variables

```
Interpretation 1. Interpret the main effects

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Estimate</th>
<th>Std. Error</th>
<th>df</th>
<th>t</th>
<th>Sig</th>
<th>95% Confidence Interval</th>
</tr>
</thead>
<tbody>
<tr>
<td>Intercept</td>
<td>-253255</td>
<td>0.171503</td>
<td>17000</td>
<td>-1.476</td>
<td>.158</td>
<td>-815150</td>
</tr>
<tr>
<td>log(GDP)</td>
<td>0.201424</td>
<td>0.021816</td>
<td>29922.404</td>
<td>13.066</td>
<td>.000</td>
<td>.239252</td>
</tr>
<tr>
<td>log(education)</td>
<td>0.046108</td>
<td>0.010384</td>
<td>0</td>
<td>0</td>
<td>.000</td>
<td>.028981</td>
</tr>
<tr>
<td>log(BNPPPS_centr)</td>
<td>0.091523</td>
<td>0.034068</td>
<td>18.167</td>
<td>14.856</td>
<td>.000</td>
<td>.449825</td>
</tr>
<tr>
<td>log(BNPPPS_centr)*log(education)</td>
<td>0.045044</td>
<td>0.043777</td>
<td>16.949</td>
<td>6.067</td>
<td>.000</td>
<td>.017220</td>
</tr>
<tr>
<td>log(BNPPPS_centr)*log(GDP)</td>
<td>0.034469</td>
<td>0.045044</td>
<td>18.167</td>
<td>-2.048</td>
<td>.051</td>
<td>-0.199546</td>
</tr>
<tr>
<td>log(education)*log(BNPPPS_centr)</td>
<td>0.000000</td>
<td>10.110</td>
<td>0.000</td>
<td>0.000</td>
<td>.000</td>
<td>-0.001902</td>
</tr>
</tbody>
</table>
```

- This parameter is set to zero because it is redundant.
- Dependent variable: Income on country level.

**Main effect education:** When BNPPPS_centr is equal to 0, someone’s income will increase with 0.5192 scale points when his educational level increases with one unit.

**Main effect GDP:** When education is equal to 0, someone’s income will increase with 0.0265 scale points when his countries BNP_PPS increases with one unit.
STEP 9 Testing cross-level interactions

Interpretation 1. Interpret the main effects

<table>
<thead>
<tr>
<th>Parameter</th>
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<th>Std. Error</th>
<th>t</th>
<th>Sig.</th>
<th>95% Confidence Interval</th>
</tr>
</thead>
<tbody>
<tr>
<td>Intercept</td>
<td>-2.53256</td>
<td>.171599</td>
<td>17.090</td>
<td>.000</td>
<td>-2.91738</td>
</tr>
<tr>
<td>[gpa]*[educ-100]</td>
<td>.281424</td>
<td>.021916</td>
<td>13.020</td>
<td>.000</td>
<td>.238252</td>
</tr>
<tr>
<td>[gpa]*[educ-100]</td>
<td>0</td>
<td>.000000</td>
<td>0.000</td>
<td>.999</td>
<td>-1.08926</td>
</tr>
<tr>
<td>bnlppps_centr</td>
<td>.113237</td>
<td>.024894</td>
<td>4.586</td>
<td>.000</td>
<td>.063360</td>
</tr>
<tr>
<td>bnlppps_centr</td>
<td>.029299</td>
<td>.004277</td>
<td>6.799</td>
<td>.000</td>
<td>.019732</td>
</tr>
<tr>
<td>bnlppps_centr</td>
<td>.036481</td>
<td>.045044</td>
<td>0.808</td>
<td>.416</td>
<td>.113896</td>
</tr>
<tr>
<td>bnlppps_centr*bnlppps_centr</td>
<td>4.439444</td>
<td>.000680</td>
<td>16.110</td>
<td>.000</td>
<td>.000183</td>
</tr>
</tbody>
</table>

Because of the centering (mean = 0) we can also say:

Main effect education: At a mean BNLPPPS_centr level, someone's income will increase with 0.1132 scale points when his educational level increases with one unit.

Main effect GDP: At a mean educational level, someone's income will increase with 0.0365 scale points when his countries BNLPPPS increases with one unit.

STEP 9 Testing cross-level interactions

Interpretation 2. Choose an interpretation perspective

2 perspectives:

1. How does the effect of education on income changes for different levels of BNLPPPS?

2. How does the effect of BNLPPPS on income changes for different levels of education?
STEP 9 Testing cross-level interactions

Interpretation 2. Choose an interpretation perspective

"How does the effect of education on income changes for different levels of BNP_PPS?" is theoretically the only possibility here.

BECAUSE: BNP_PPS will not change if people have a higher degree. But the educational effects can indeed change for countries with different BNP_PPS.

BUT REMEMBER: Statistically, both perspectives are always possible.

STEP 9 Testing cross-level interactions

Interpretation 3. Work out the regression equation

PERSPECTIVE: How does the effect of education on income changes for different levels of BNP_PPS?

- Write out the regression equation for education, when BNP_PPS takes on different levels.
- WHICH levels of BNP_PPS?
  Usually: minimum, 25th percentile, median, mean, 75th percentile and maximum.
STEP 9 Testing cross-level interactions

Interpretation 3. Work out the regression equation

STEP 1: Search in the EXAMINE output for the values of the minimum, 25th percentile, median, mean, 75th percentile and the maximum of BNPPPS_centr.

```
EXAMINE VARIABLES=bnppps_centr
/PERCENTILES(25,75)
/STATISTICS DESCRIPTIVES.
```
STEP 9 Testing cross-level interactions

Interpretation 3. Work out the regression equation

STEP 1

Minimum  -65.8815
Q1       -23.2815
Mean     0
Median   2.1185
Q3       14.6185
maximum  126.9125

Remark: We assume that the other independent variable also take on the value of 0 (and disappear from the equation).

Income = -0.2532 + 0.5192 education + 0.0265 BNPPPS + 0.0008 EDUC*BNPPPS

STEP 2: Vul in

Minimum Income = -0.2532 + 0.5192 education + 0.0265 (-65.88) + 0.0008 OPL*(-65.88)
Q1 Income = -0.2532 + 0.5192 education + 0.0265 (-23.2815) + 0.0008 OPL*(-23.2815)
Median Income = -0.2532 + 0.5192 education + 0.0265 (0) + 0.0008 OPL*(0)
Q3 Income = -0.2532 + 0.5192 education + 0.0265 (14.6185) + 0.0008 OPL*(14.6185)
maximum Income = -0.2532 + 0.5192 education + 0.0265 (126.9125) + 0.0008 OPL*(126.9125)
STEP 9 Testing cross-level interactions

Interpretation 3. Work out the regression equation

\[
\text{Income} = -0.2532 + 0.5192 \text{education} + 0.0265 \text{BNPPPS} + 0.0008 \text{EDUC*BNPPPS}
\]

STEP 2: Vul in

Minimum Income = -1,999 + 0.4665 OPL
Q1 Income = -0.8702 + 0.5006 OPL
Mean Income = -0.1971 + 0.5209 OPL
Q3 Income = 0.1342 + 0.5309 OPL
Maximum Income = 3.11 + 0.6207 OPL

Interpretation interaction effect

The effect of education rises (very slightly) as BNP_PPS increases.

STEP 10 Comparing models with Full Maximum Likelihood

- Estimation in multilevel analysis
  - Maximum Likelihood
  - Generalized Least Squares (not in SPSS)
  - Generalized Estimation Equations (not in SPSS)
  - Markov Chain Monte Carlo (not in SPSS)
  - Bootstrapping (not in SPSS)
STEP 10 Comparing models with Full Maximum Likelihood

- **Maximum Likelihood**
  - Maximizing the likelihood function
  - Population parameters are estimated in such a way that the probability of finding that particular model is maximized with the data at hand.
  - **TWO versions:** Full Maximum Likelihood (ML) Restricted Maximum Likelihood (REML)

- **Restricted Maximum Likelihood**
  - Only the variance components are used in the estimation.
  - Standard in SAS (but not in SPSS) and in almost all multilevel programs
  - Best estimation for the variance components
  - **CONSEQUENCE:** Gives you the best estimation for the parameters and you should use this estimation method to obtain your parameters
STEP 10 Comparing models with Full Maximum Likelihood

- **Full Maximum Likelihood**
  - Variance components and regression coefficients are involved in the estimation.
  - Disadvantage: Variance components are underestimated.

USEFUL?
- Easier estimation method
- The difference between two models can be used in a \( \chi^2 \)-difference test (is not possible with REML!).

STEP 10 Comparing models with Full Maximum Likelihood

- **CONSEQUENCE**
  - Use REML for all models
  - Use ML at the end to compare estimated models with each other.
STEP 10 Comparing models with Full Maximum Likelihood

* STEP 10: Using Full Maximum Likelihood to compare models.

**Step 2.**

```
MIXED ink_centr
    /PRINT = SOLUTION TESTCOV Use Method=ML (or leave the METHOD line out).
    /METHOD = ML
    /FIXED = INTERCEPT
    /RANDOM = INTERCEPT | SUBJECT(cntry2).
```

Use the difference in DEVIANCE (-2 LOG LIKELIHOOD) and the difference in degrees of freedom in a Chi²-difference test.
STEP 10 Comparing models with Full Maximum Likelihood

• Calculating the degrees of freedom of your model

Calculating the degrees of freedom by hand:

<table>
<thead>
<tr>
<th>Term</th>
<th>Degrees of Freedom</th>
</tr>
</thead>
<tbody>
<tr>
<td>Intercept</td>
<td>1</td>
</tr>
<tr>
<td>Error variance level-1</td>
<td>1</td>
</tr>
<tr>
<td>Fixed slopes on level-1</td>
<td>(p)</td>
</tr>
<tr>
<td>Error-variances for slopes on level-1</td>
<td>(p)</td>
</tr>
<tr>
<td>Co-variances of slopes and intercept</td>
<td>(p)</td>
</tr>
<tr>
<td>Co-variances between the slopes</td>
<td>(p(p-1)/2)</td>
</tr>
<tr>
<td>Fixed slopes on level-2</td>
<td>(q)</td>
</tr>
<tr>
<td>Slopes for cross-level interactions</td>
<td>(p^2q)</td>
</tr>
</tbody>
</table>

Whereby:

\[ p = \text{number of level-1 predictors} \]

\[ q = \text{number of level-2 predictors} \]

Make an overview table with tests

<table>
<thead>
<tr>
<th>Step</th>
<th>Model Description</th>
<th>Deviance</th>
<th>(D(f))</th>
<th>Prob. Chi² vs null model</th>
<th>Prob. Chi² vs previous model</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>Null random intercept model</td>
<td>141642,268</td>
<td>2</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>Random intercept with level-1</td>
<td>136763,937</td>
<td>9</td>
<td>0,000</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>Random intercept with level-1 and 2</td>
<td>122423,421</td>
<td>13</td>
<td>0,000</td>
<td>0,000</td>
</tr>
<tr>
<td>6</td>
<td>Random slopes model</td>
<td>136407,950</td>
<td>9</td>
<td>0,000</td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>Random slopes with level-2</td>
<td>122075,321</td>
<td>11</td>
<td>0,000</td>
<td>0,000</td>
</tr>
<tr>
<td>8     Random slopes with level-2 (reduction)</td>
<td>122077,357</td>
<td>7</td>
<td>0,000</td>
<td>0,729</td>
<td></td>
</tr>
<tr>
<td>9     Cross level interactions</td>
<td>122077,357</td>
<td>12</td>
<td>0,000</td>
<td>1,000</td>
<td></td>
</tr>
</tbody>
</table>
STEP 10 Comparing models with Full Maximum Likelihood

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<td>11</td>
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</tr>
<tr>
<td>9</td>
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<td>122077.357</td>
<td>12</td>
<td>0.000</td>
<td>1.000</td>
</tr>
</tbody>
</table>

Chi²-difference test
- Test value = 141642.268 – 136763.937 = 4878
- Degrees of freedom = 9 – 2 = 7
- Result: Probability = 0.000

Interpretation: The model in STEP3 is a significant improvement of the Null random intercept model (STEP 2)

Comparison with the Null random intercept model (STEP 2)
Interpretation: Is this model a significant improvement vs STEP 2?

Comparison with the previous model (= STEP 3)
Interpretation: Is this model a significant improvement vs STEP 3?
STEP 10 Comparing models with Full Maximum Likelihood

Make an overview table with tests

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<td>122075.321</td>
<td>11</td>
<td>0.000</td>
<td>0.000</td>
</tr>
</tbody>
</table>

BE CAREFUL here!!

Previous model is the model with co-variances. SO is the model without co-variances a significant worsening vs. the previous model?

CONSEQUENCE: A probability higher than 0.05 points to a good model.

STEP 11 The final model with standardized variables

- Standard output of multilevel models = not standardized.
- Interpretation in original (scale) units
- DISADVANTAGE: no comparison of the actual size of the parameters.
STEP 11 The final model with standardized variables

- **2 methods**

1. **Estimate the last model again with standardized variables**

   (disadvantage = variance components also change)

2. **Standardize the parameter by hand**

---

**METHOD 1**

```
* STEP 11: Standardising in order to compare parameters. *

MIXED ink_std BY gesl_std WITH opl_std bnppps_std romkath_std
/PRINT = SOLUTION TESTCOV
/METHOD = ML
/FIXED = INTERCEPT geslacht opl_std bnppps_std romkath_std
/RANDOM = INTERCEPT opl_std | SUBJECT(cntry2).
```
METHOD 2

\[
\text{Stand. Coeff.} = \frac{(\text{non stand. Coeff}) \times (\text{stan dev INdep.})}{(\text{stan dev DEpendent})}
\]

**STEP 11 The final model with standardized variables**

**METHOD 2**

DESCRIPTIVES VARIABLES=ink_centr geslacht opl_centr bnppps_centr romkath_centr /STATISTICS=STDDEV.
### STEP 11 The final model with standardized variables

**RESULT:**

<table>
<thead>
<tr>
<th>Variable</th>
<th>Non-standardized</th>
<th>Standardized</th>
<th>Non-standardized</th>
<th>Standardized</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gender</td>
<td>0.281</td>
<td>0.06</td>
<td>BNP PPS</td>
<td>0.027</td>
</tr>
<tr>
<td>Education</td>
<td>0.519</td>
<td>0.31</td>
<td>% Rom Cath</td>
<td>-0.095</td>
</tr>
</tbody>
</table>

---

**MULTILEVEL MODELS**

Multilevel-analysis in SPSS - step by step

Dimitri Mortelmans  
Centre for Longitudinal and Life Course Studies (CLLS)  
University of Antwerp